

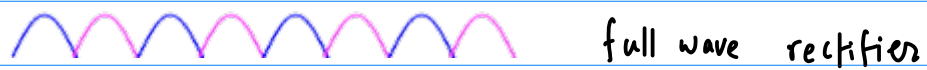
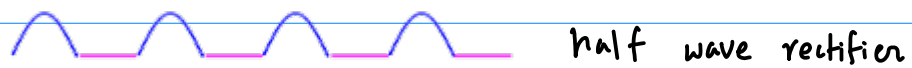
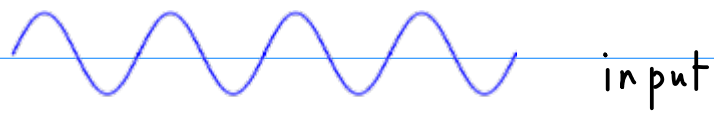
Rectifier (H.1)

20170407

Copyright (c) 2017 Young W. Lim.

Permission is granted to copy, distribute and/or modify this document under the terms of the GNU Free Documentation License, Version 1.2 or any later version published by the Free Software Foundation; with no Invariant Sections, no Front-Cover Texts, and no Back-Cover Texts. A copy of the license is included in the section entitled "GNU Free Documentation License".

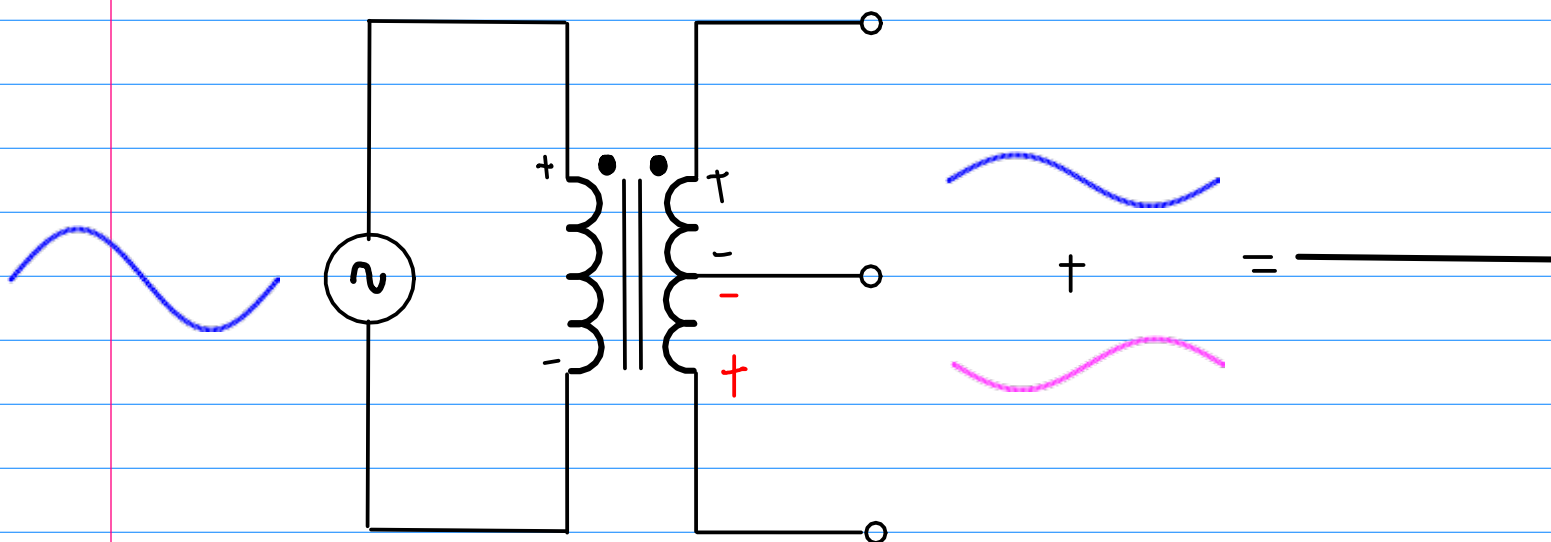
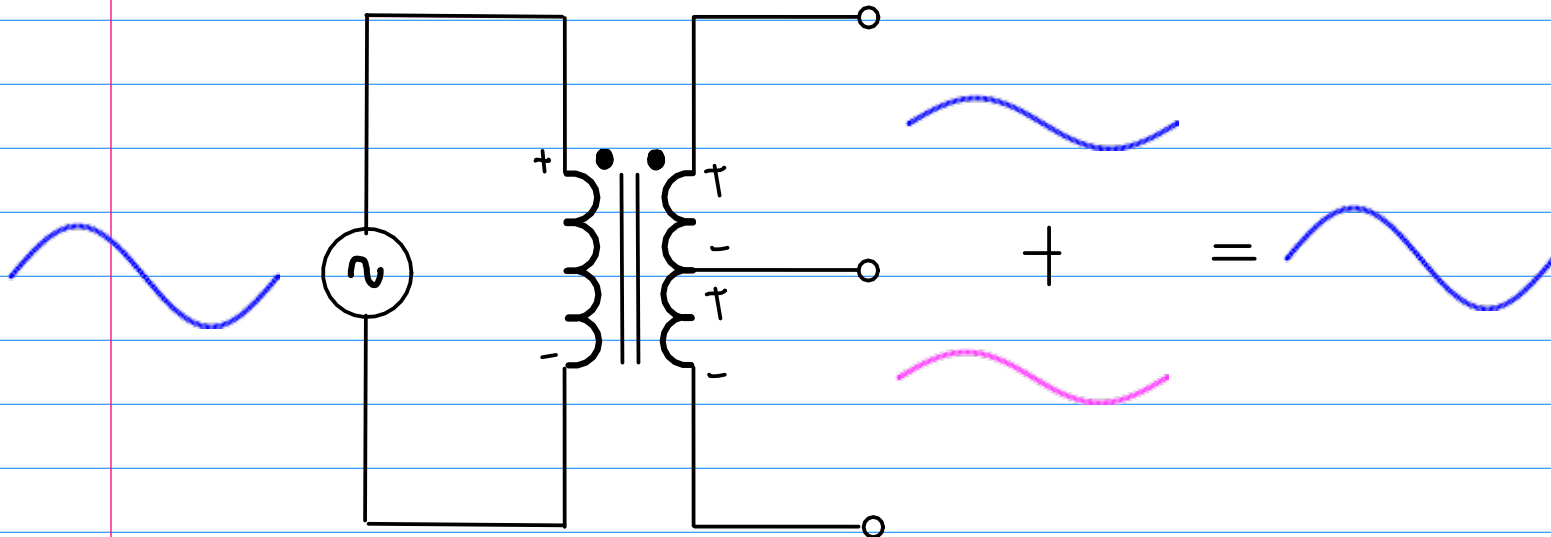
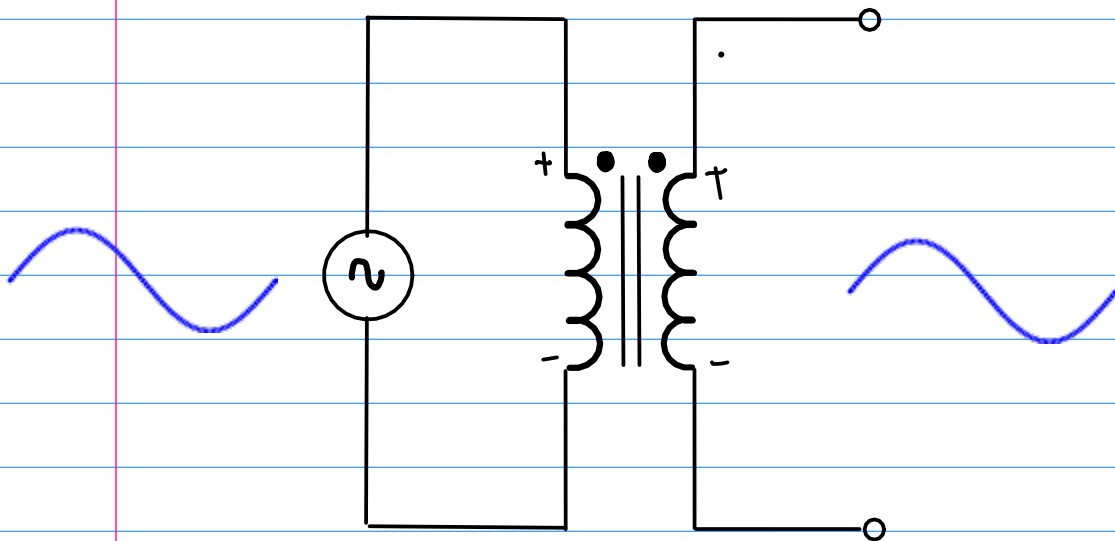
Rectifiers



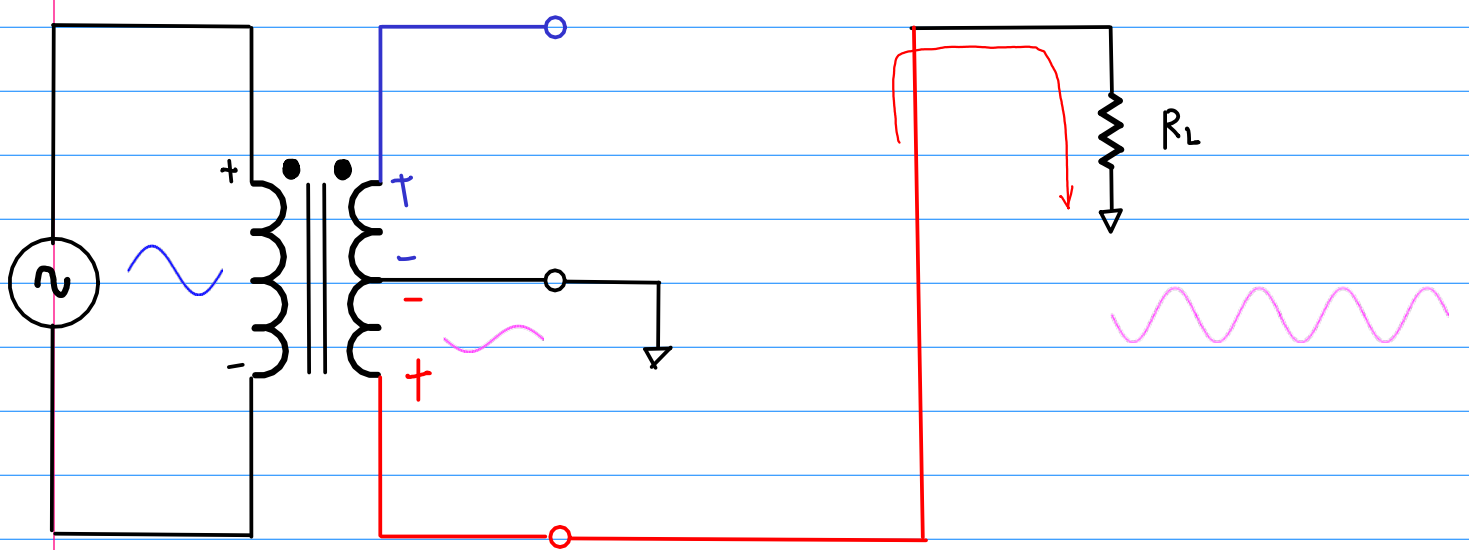
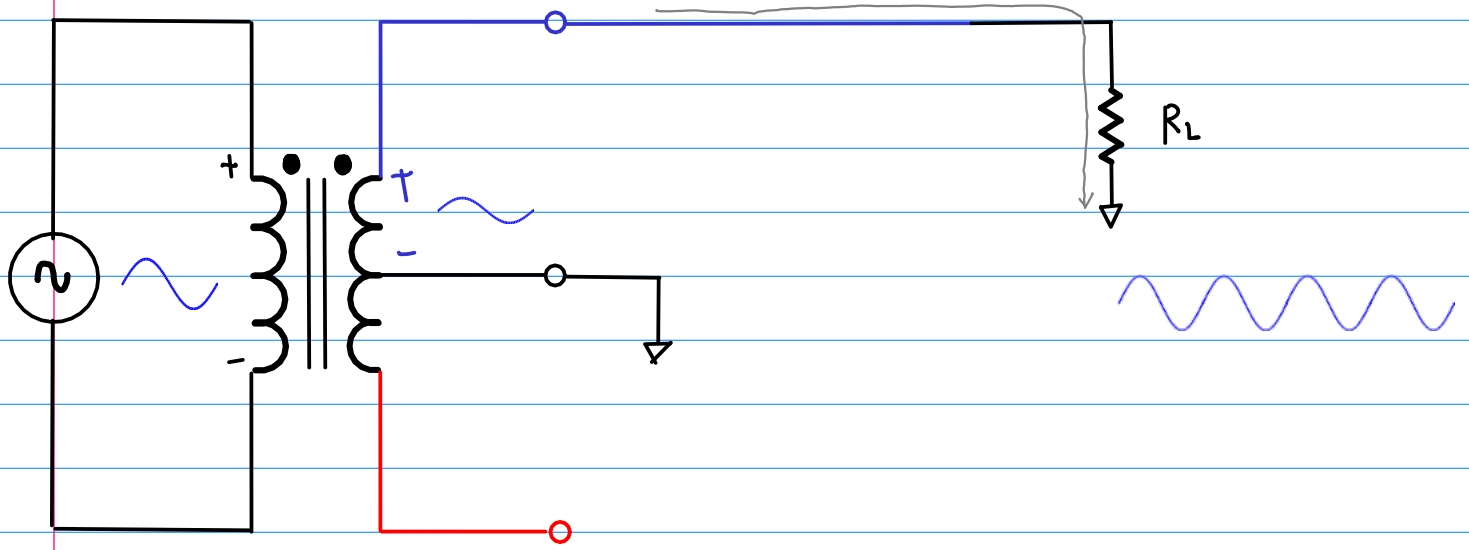
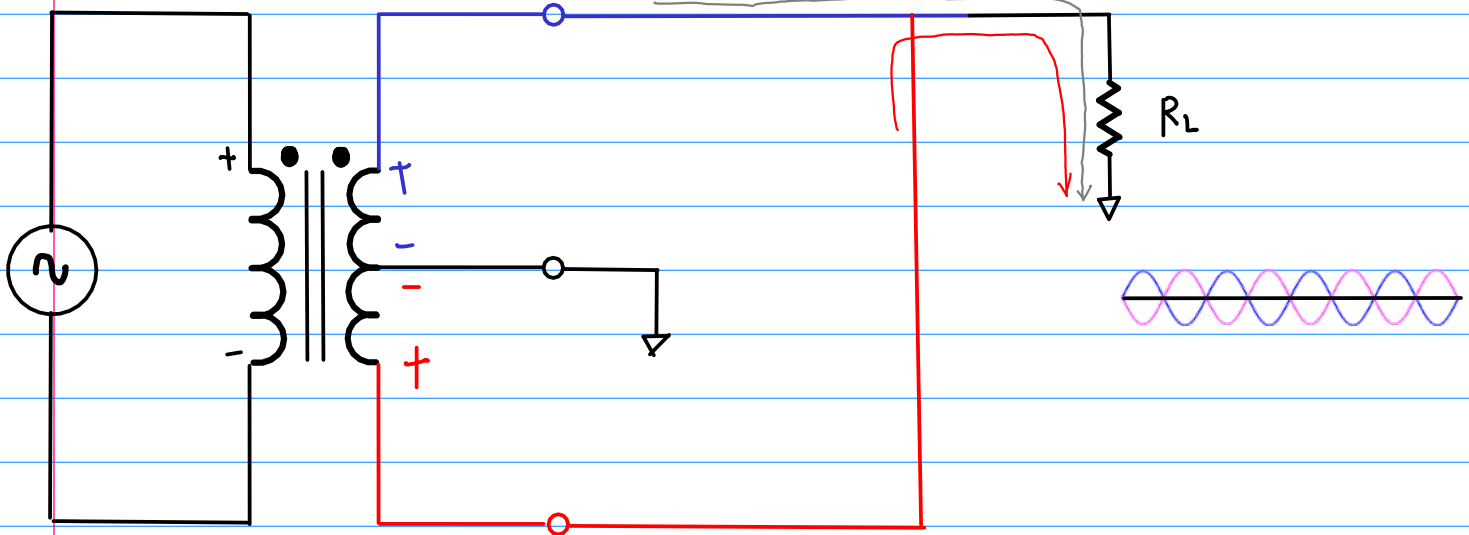
① Tapped Transformer

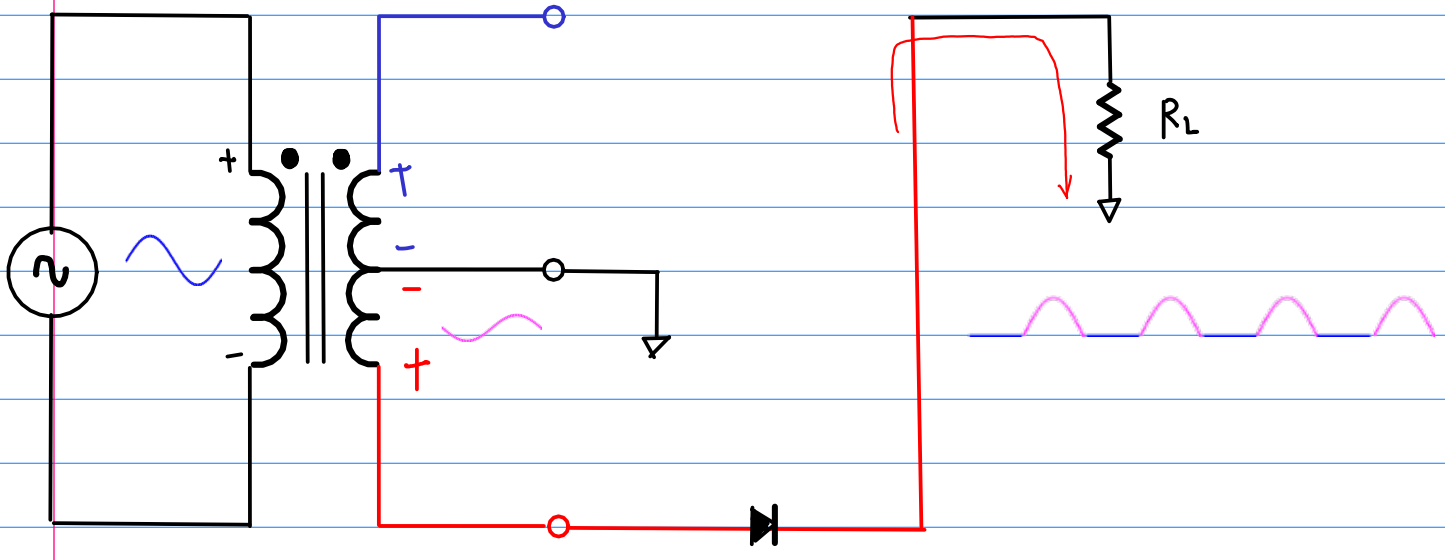
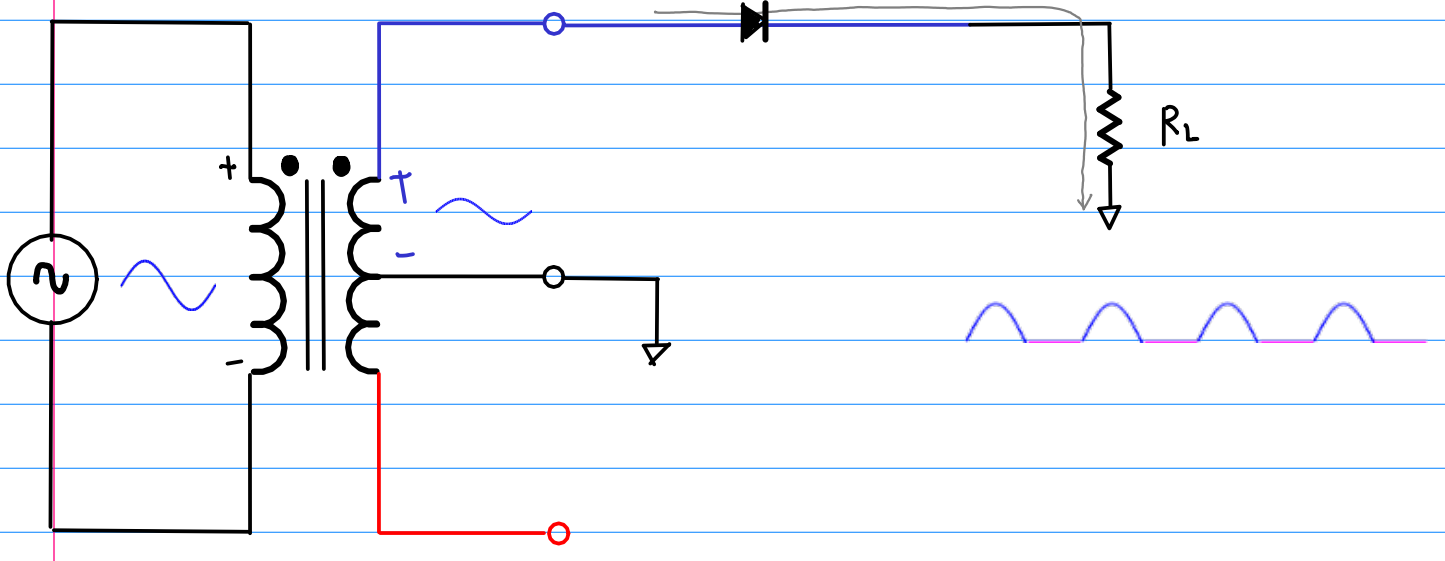
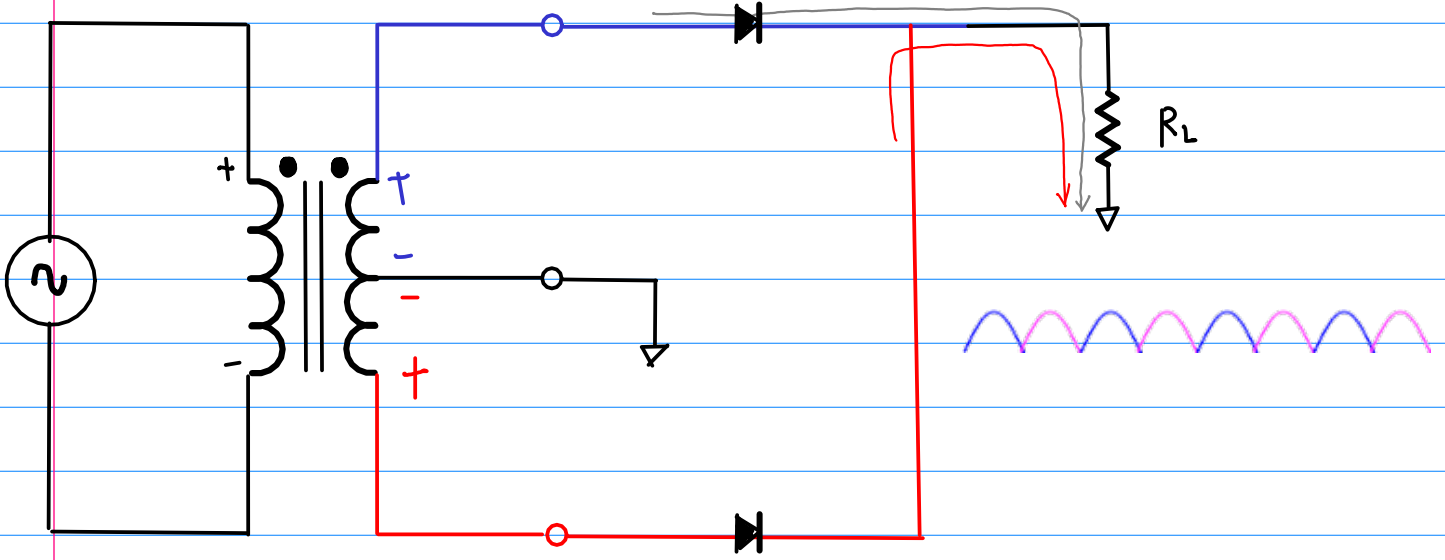
② Bridge

Tapped Transformer

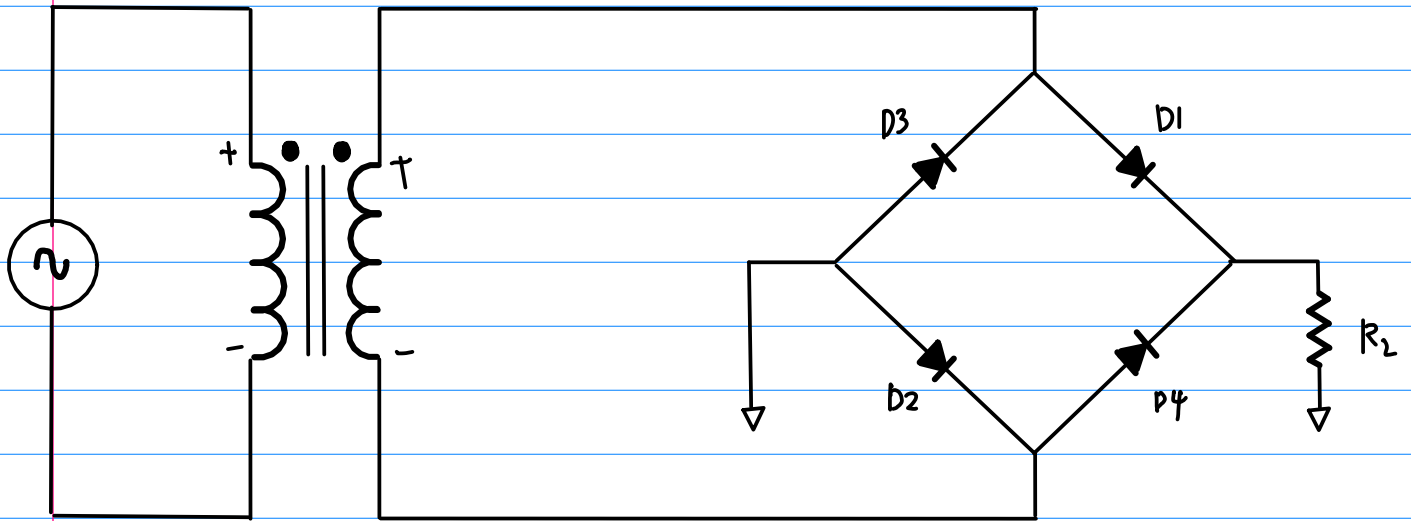


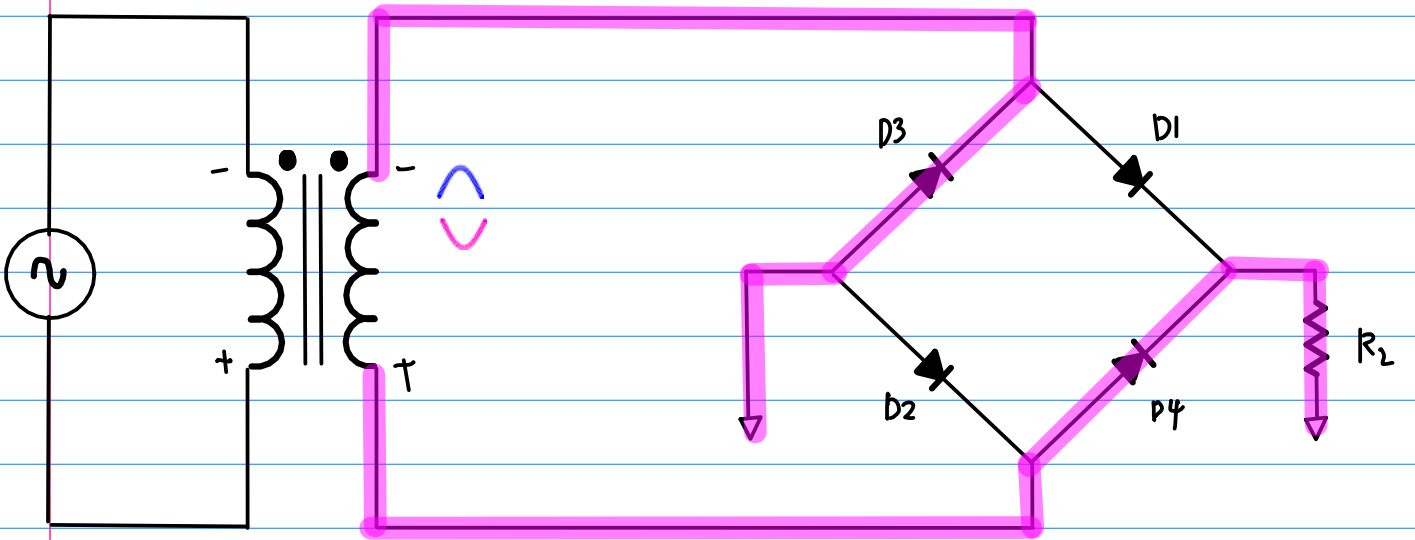
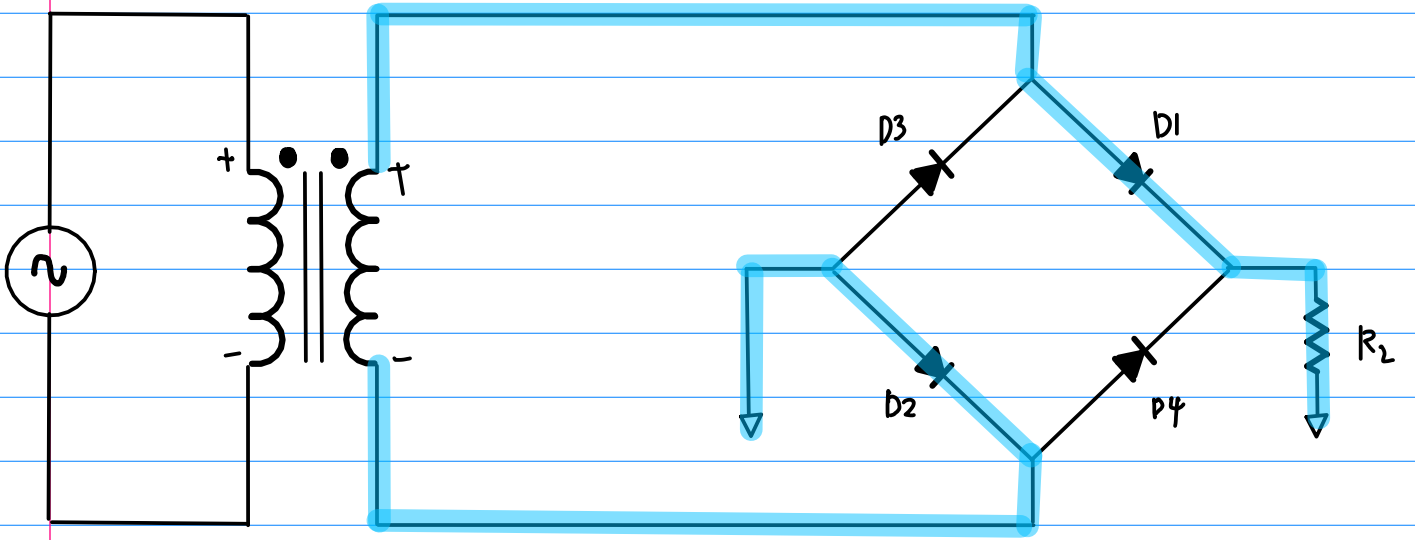
Supper position

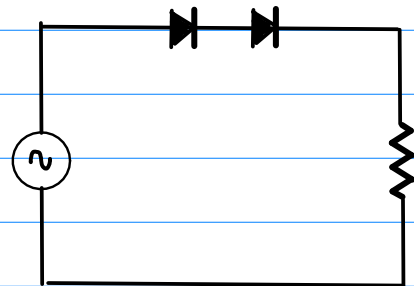
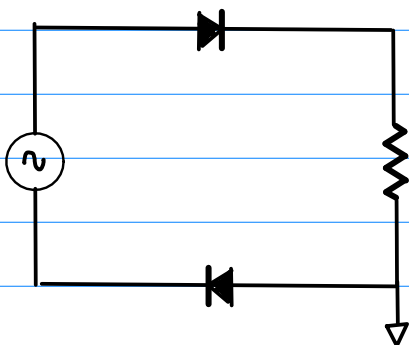
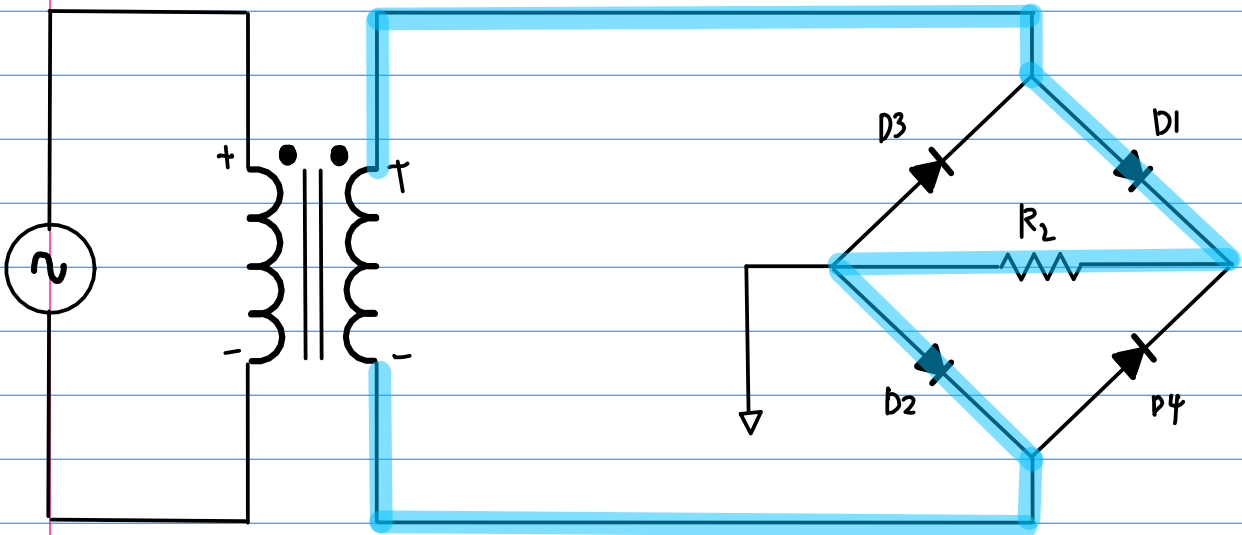
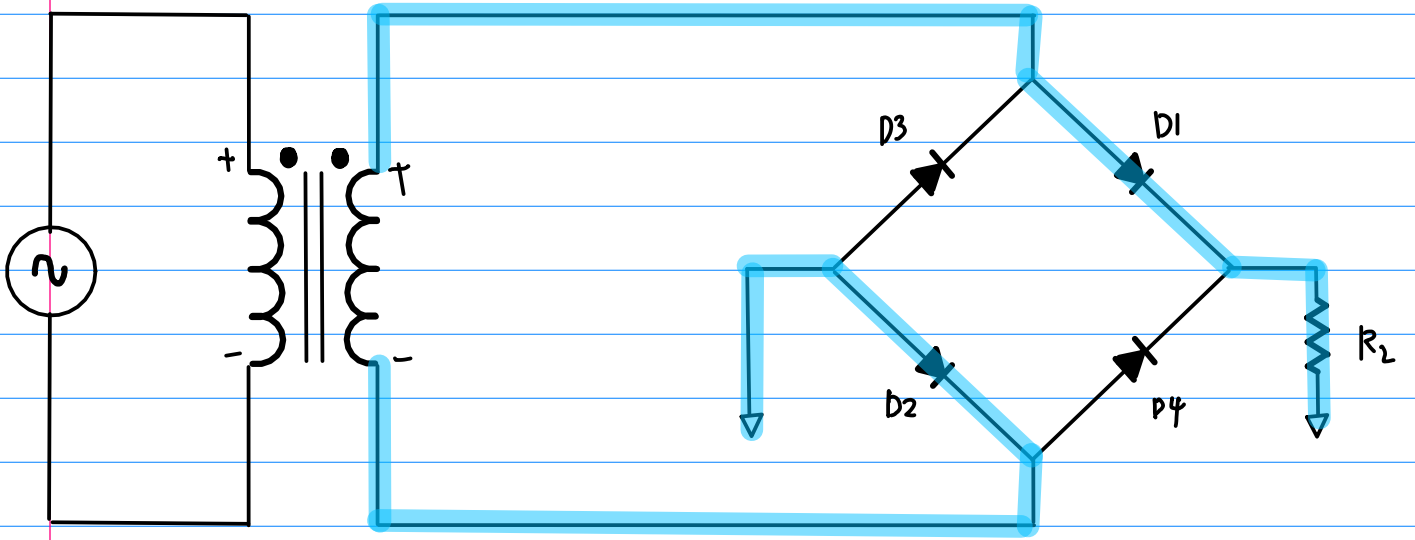


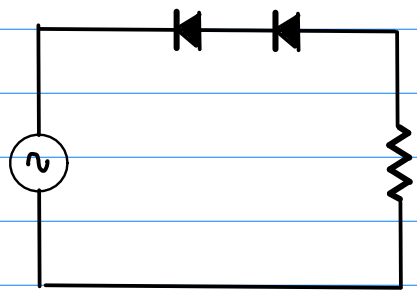
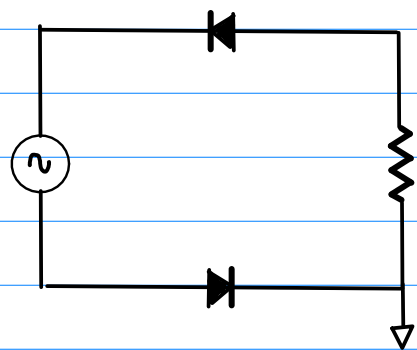
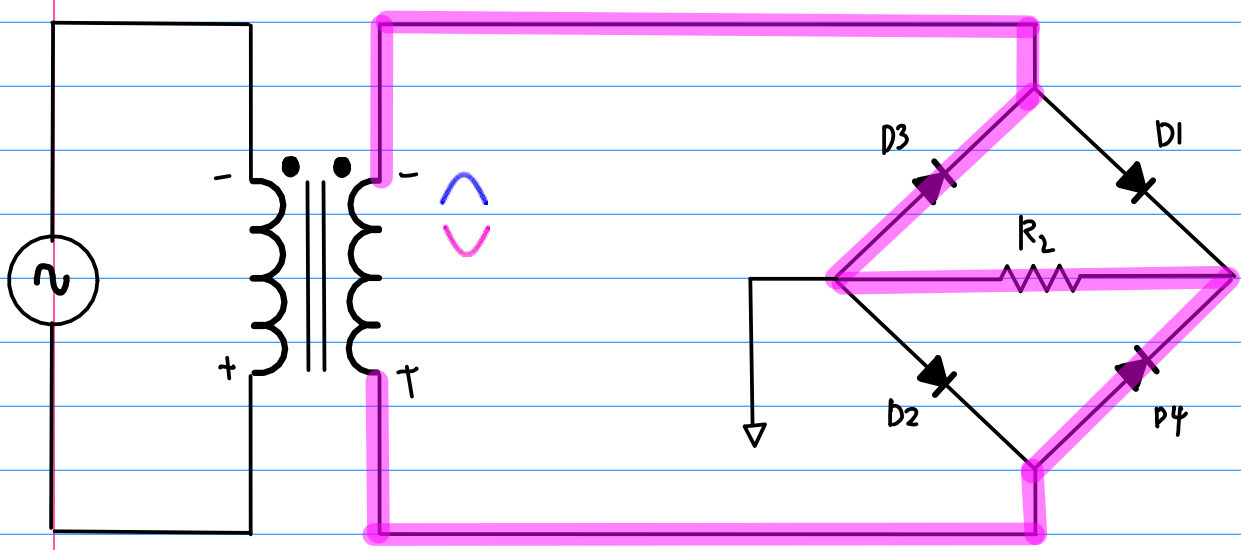
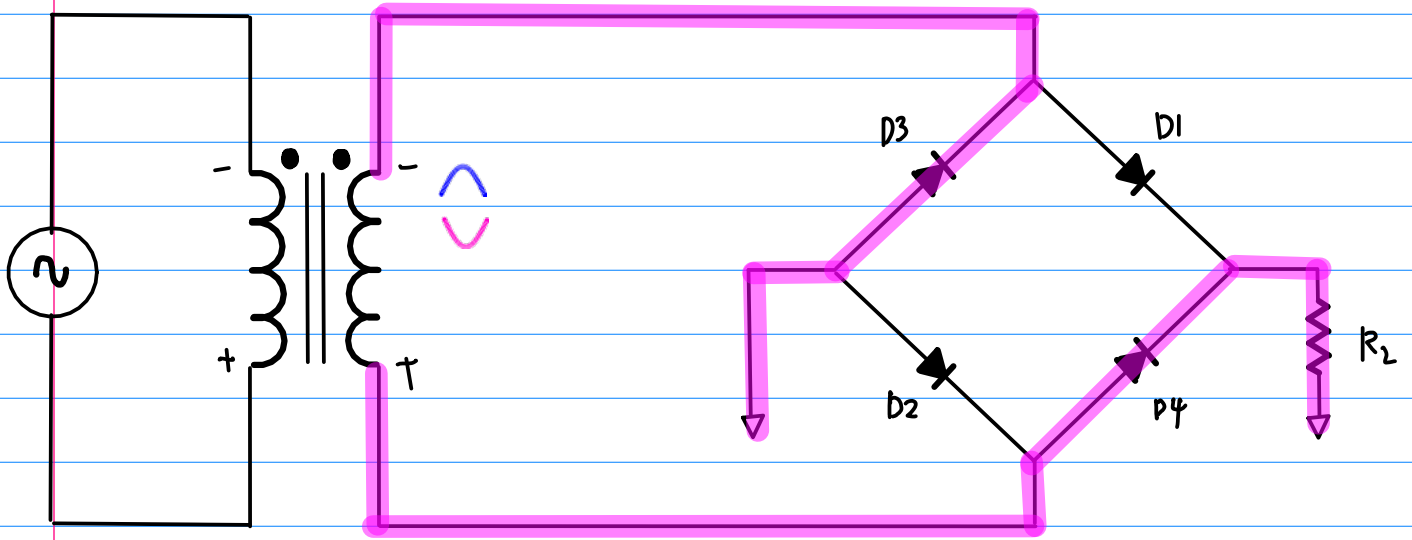


Bridge Rectifier

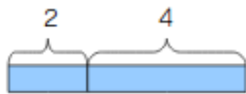








Arithmetic Mean – Example



- same length

$$(2+4)=(3+3)=2 \cdot 3=2 \cdot A$$

Arithmetic Mean: $A = 3$



- same length

$$(2+4+6)=(4+4+4)=3 \cdot 4=3 \cdot A$$

Arithmetic Mean: $A = 4$

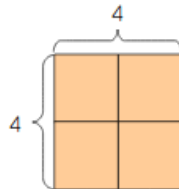
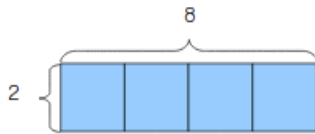
Geometric Mean

2 elements $\{a, b\}$ $G = \sqrt{a \cdot b}$ ($a > 0, b > 0$)

3 elements $\{a, b, c\}$ $G = \sqrt[3]{a \cdot b \cdot c}$ ($a > 0, b > 0, c > 0$)

n elements $\{a_1, a_2, \dots, a_n\}$ $G = \sqrt[n]{a_1 \cdot a_2 \cdots a_n} = \left(\prod_{i=1}^n a_i\right)^{\frac{1}{n}}$ ($a_i > 0$)

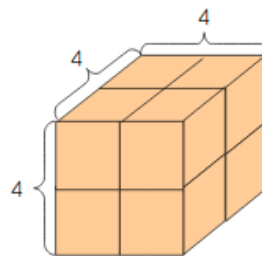
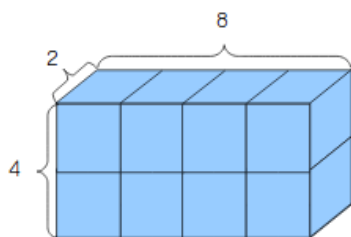
Geometric Mean – Example



$$(2 \cdot 8) = (4 \cdot 4) = 4^2 = G^2$$

- same area

Geometric Mean: $G = 4$



$$(2 \cdot 4 \cdot 8) = (4 \cdot 4 \cdot 4) = 4^3 = G^3$$

- same volume

Geometric Mean: $G = 4$

Root Mean Square

The RMS value of a set of values (or a **continuous-time waveform**) is the square root of the arithmetic mean of the squares of the values, or the square of the function that defines the continuous waveform. In Physics, the rms current is the "value of the direct current that dissipates power in a resistor."

In the case of a set of n values $\{x_1, x_2, \dots, x_n\}$, the RMS

$$x_{\text{rms}} = \sqrt{\frac{1}{n} (x_1^2 + x_2^2 + \dots + x_n^2)}.$$

The corresponding formula for a continuous function (or waveform) $f(t)$ defined over the interval $T_1 \leq t \leq T_2$ is

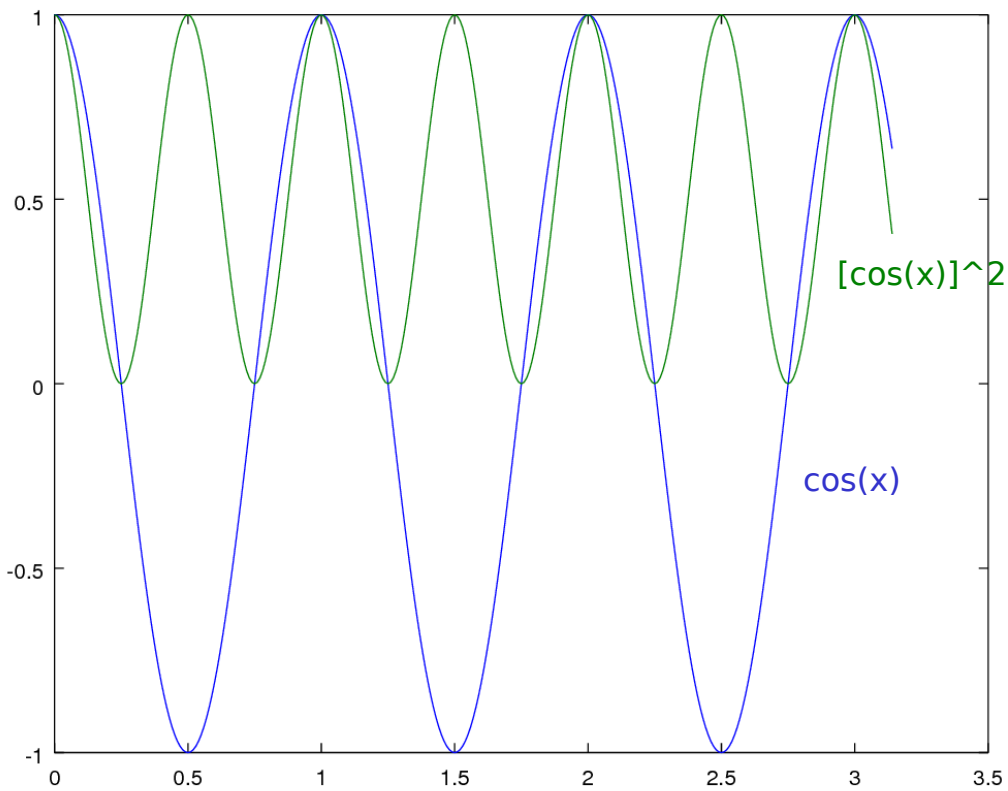
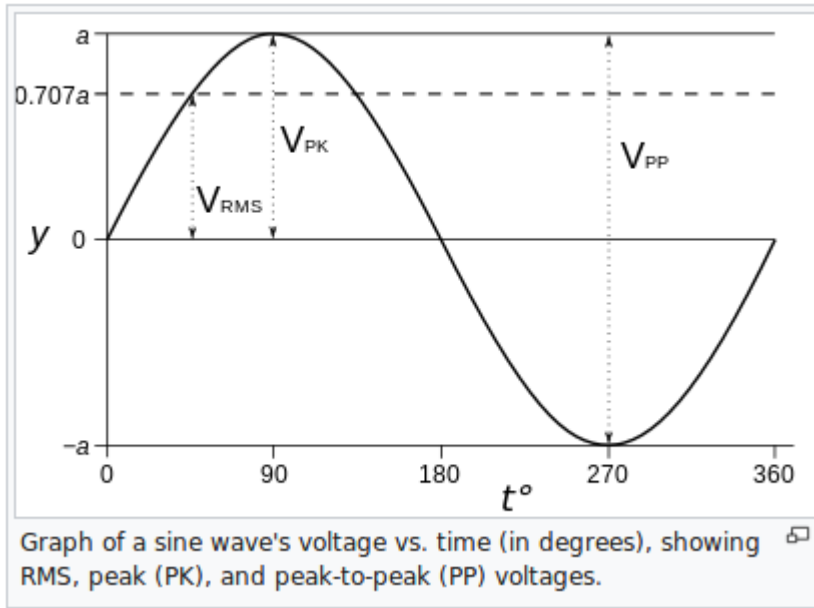
$$f_{\text{rms}} = \sqrt{\frac{1}{T_2 - T_1} \int_{T_1}^{T_2} [f(t)]^2 dt},$$

and the RMS for a function over all time is

$$f_{\text{rms}} = \lim_{T \rightarrow \infty} \sqrt{\frac{1}{T} \int_0^T [f(t)]^2 dt}.$$

The RMS over all time of a **periodic function** is equal to the RMS of one period of the function. The RMS value of a continuous function or signal can be approximated by taking the RMS of a sequence of equally spaced samples. Additionally, the RMS value of various waveforms can also be determined without **calculus**, as shown by Cartwright.^[2]

In the case of the RMS statistic of a **random process**, the **expected value** is used instead of the mean.



Square

$$\cos^2(x)$$

mean

$$\frac{1}{2\pi} \int_0^{2\pi} \cos^2(x) dx$$

root

$$\sqrt{\frac{1}{2\pi} \int_0^{2\pi} \cos^2(x) dx}$$

```
(%i2) (cos(x))^2;
(%o2) cos(x)^2
```

```
(%i3) trigreduce(%);
(%o3)  $\frac{1+\cos(2x)}{2}$ 
```

```
-->
```

```
(%i4) integrate(%, x);
(%o4)  $x + \frac{\sin(2x)}{2}$ 
```

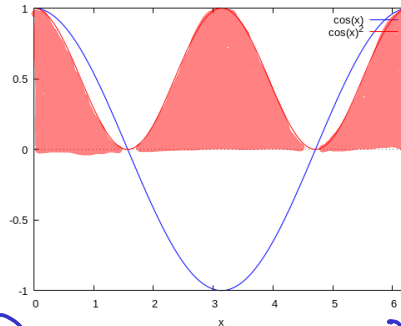
```
-->
```

```
(%i5) integrate((1+cos(2*x))/2, x, 0, 2*pi);
(%o5)  $\pi$ 
```

```
(%i6) %pi/(2*pi);
(%o6)  $\frac{\pi}{2\pi}$ 
```

```
(%i7) sqrt(1/2);
(%o7)  $\frac{1}{\sqrt{2}}$ 
```

```
(%i8) float(%);
(%o8) 0.7071067811865475
```



area = π

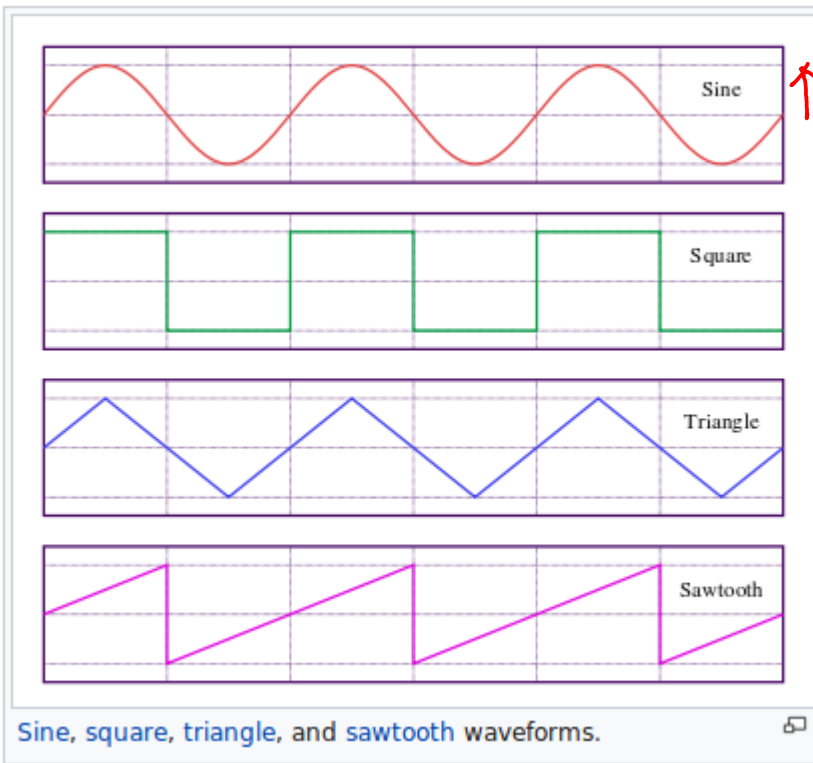
$$0.707^2 = 0.5 \Rightarrow \left(\frac{1}{2}\right) \quad 2\pi$$

area = π

$$\int_0^{2\pi} \cos^2(x) dx = \int_0^{2\pi} \frac{1}{2} (1 + \cos(2x)) dx = \pi$$

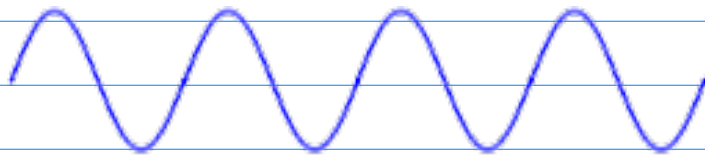
$$\frac{1}{2\pi} \int_0^{2\pi} \cos^2(x) = \frac{1}{2}$$

$$\sqrt{\frac{1}{2\pi} \int_0^{2\pi} \cos^2(x)} = \sqrt{\frac{1}{2}} = 0.707$$



Waveform	Equation	RMS
DC, constant	$y = A_0$	A_0
Sine wave	$y = A_1 \sin(2\pi ft)$	$\frac{A_1}{\sqrt{2}}$
Square wave	$y = \begin{cases} A_1 & \text{frac}(ft) < 0.5 \\ -A_1 & \text{frac}(ft) > 0.5 \end{cases}$	A_1
DC-shifted square wave	$y = A_0 + \begin{cases} A_1 & \text{frac}(ft) < 0.5 \\ -A_1 & \text{frac}(ft) > 0.5 \end{cases}$	$\sqrt{A_0^2 + A_1^2}$
Modified sine wave	$y = \begin{cases} 0 & \text{frac}(ft) < 0.25 \\ A_1 & 0.25 < \text{frac}(ft) < 0.5 \\ 0 & 0.5 < \text{frac}(ft) < 0.75 \\ -A_1 & \text{frac}(ft) > 0.75 \end{cases}$	$\frac{A_1}{\sqrt{2}}$
Triangle wave	$y = 2A_1 \text{frac}(ft) - A_1 $	$\frac{A_1}{\sqrt{3}}$
Sawtooth wave	$y = 2A_1 \text{frac}(ft) - A_1$	$\frac{A_1}{\sqrt{3}}$
Pulse train	$y = \begin{cases} A_1 & \text{frac}(ft) < D \\ 0 & \text{frac}(ft) > D \end{cases}$	$A_1 \sqrt{D}$
Phase-to-phase voltage	$y = A_1 \sin(t) - A_1 \sin\left(t - \frac{2\pi}{3}\right)$	$A_1 \sqrt{\frac{3}{2}}$

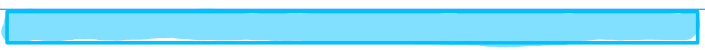
DC value of Half-Wave Signal



input



half wave rectifier



$\Leftrightarrow \frac{1}{\pi} = 0.318$



full wave rectifier

same area

area

$$f(x) = \sin(x)$$

$$\int_0^{\pi/2} \sin(t) dt = 1$$

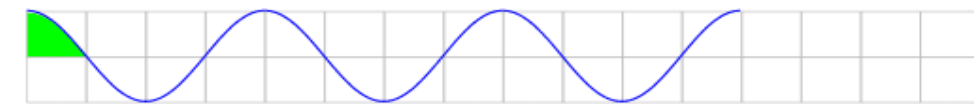
C2



$$f(x) = \cos(x)$$

$$\int_0^{\pi/2} \cos(x) dx = 1$$

D1



0 1 0 -1 0 1 0 -1 0 1 0 -1 → area - 0

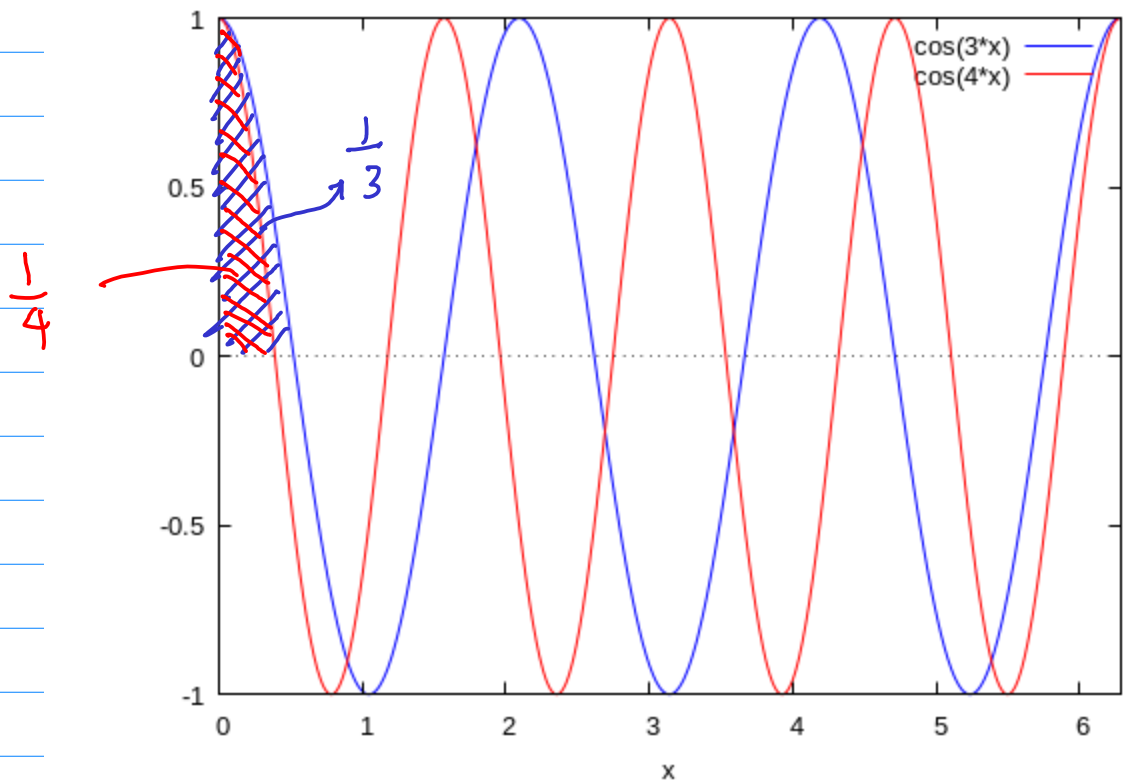
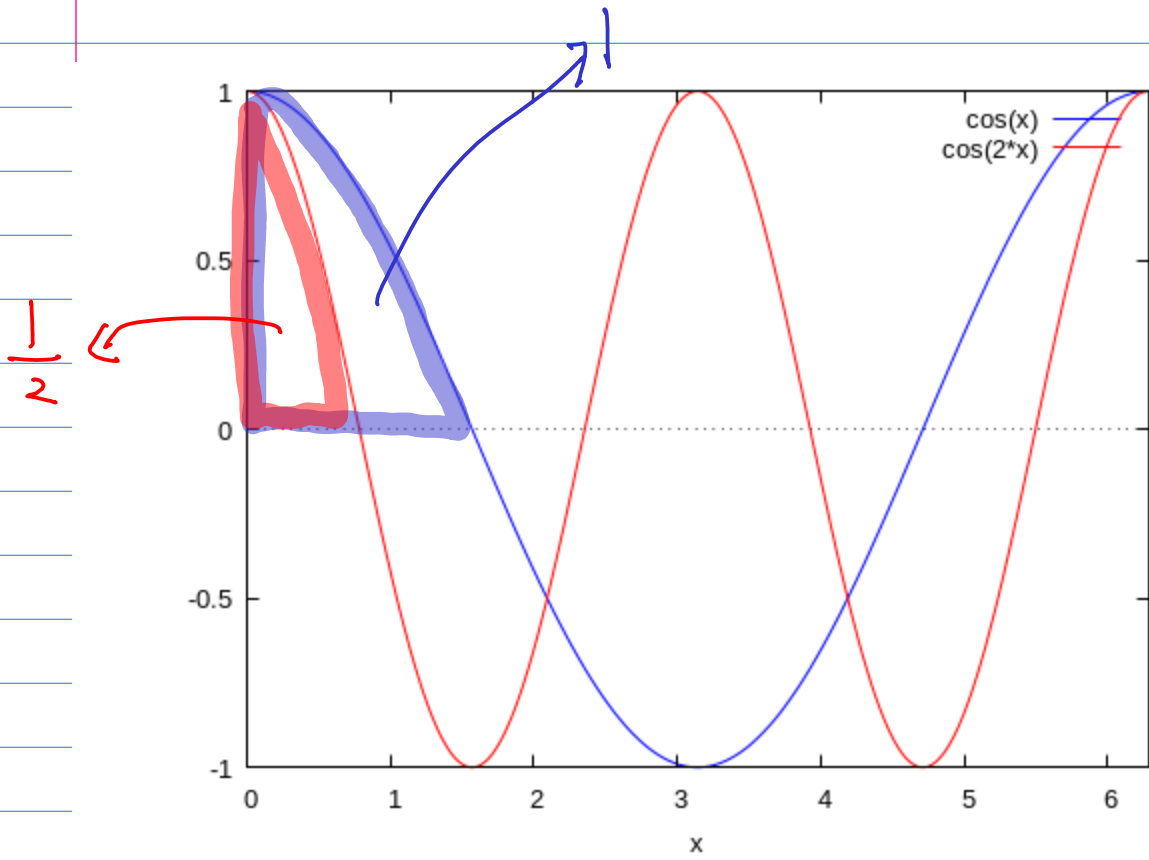
$$= \sin(x) - 0$$

```
(%i15) integrate(cos(x), x, 0, %pi/2);  
(%o15) 1
```

```
(%i3) integrate(cos(2*x), x, 0, %pi/4);  
(%o3) 1/2
```

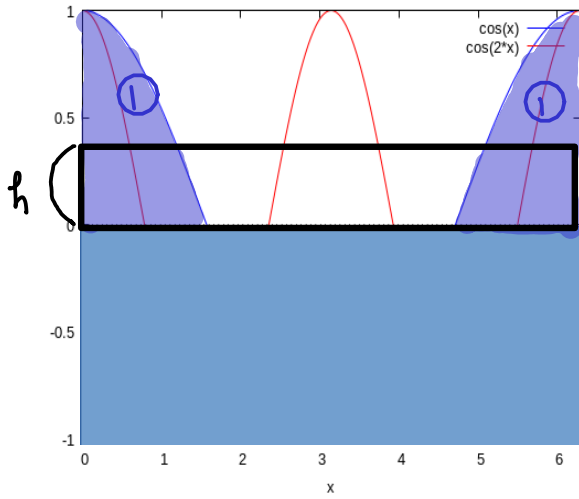
```
(%i6) integrate(cos(3*x), x, 0, %pi/6);  
(%o6) 1/3
```

```
(%i7) integrate(cos(4*x), x, 0, %pi/8);  
(%o7) 1/4
```



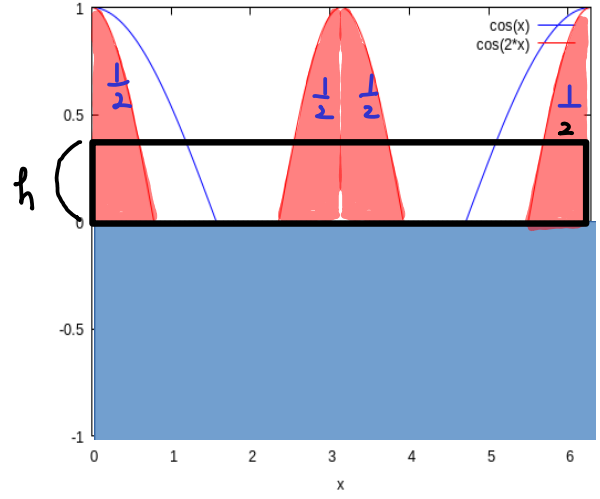
$\cos(x)$

$$1 + 1 = 2\pi h \quad h = \frac{1}{\pi}$$



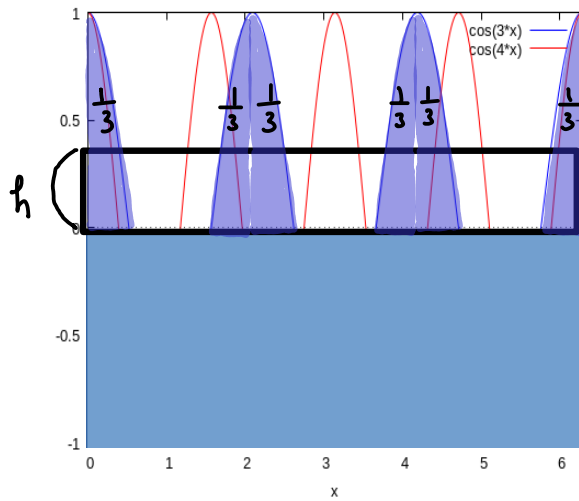
$\cos(2x)$

$$\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = 2\pi h \quad h = \frac{1}{\pi}$$



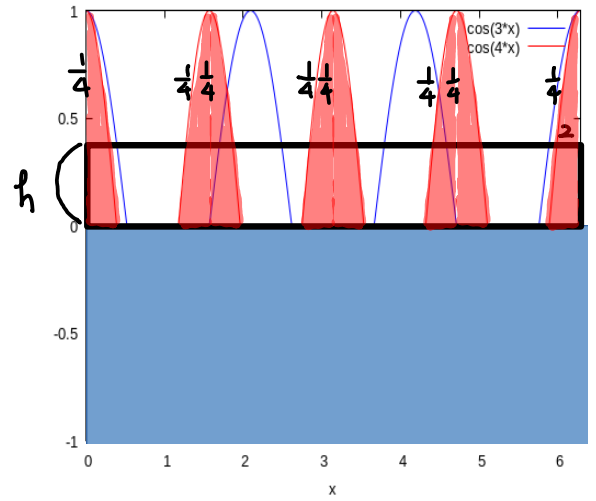
$\cos(3x)$

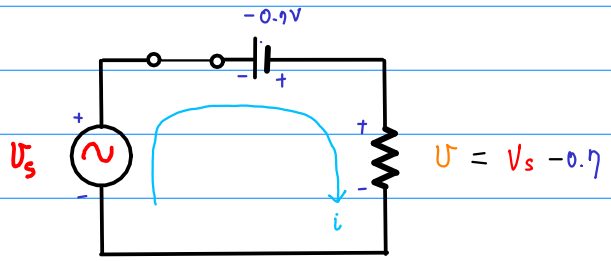
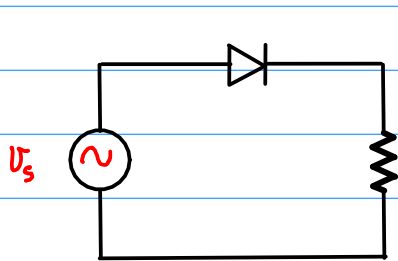
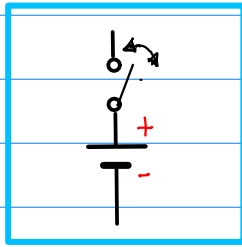
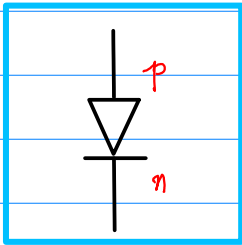
$$\frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 2\pi h \quad h = \frac{1}{\pi}$$

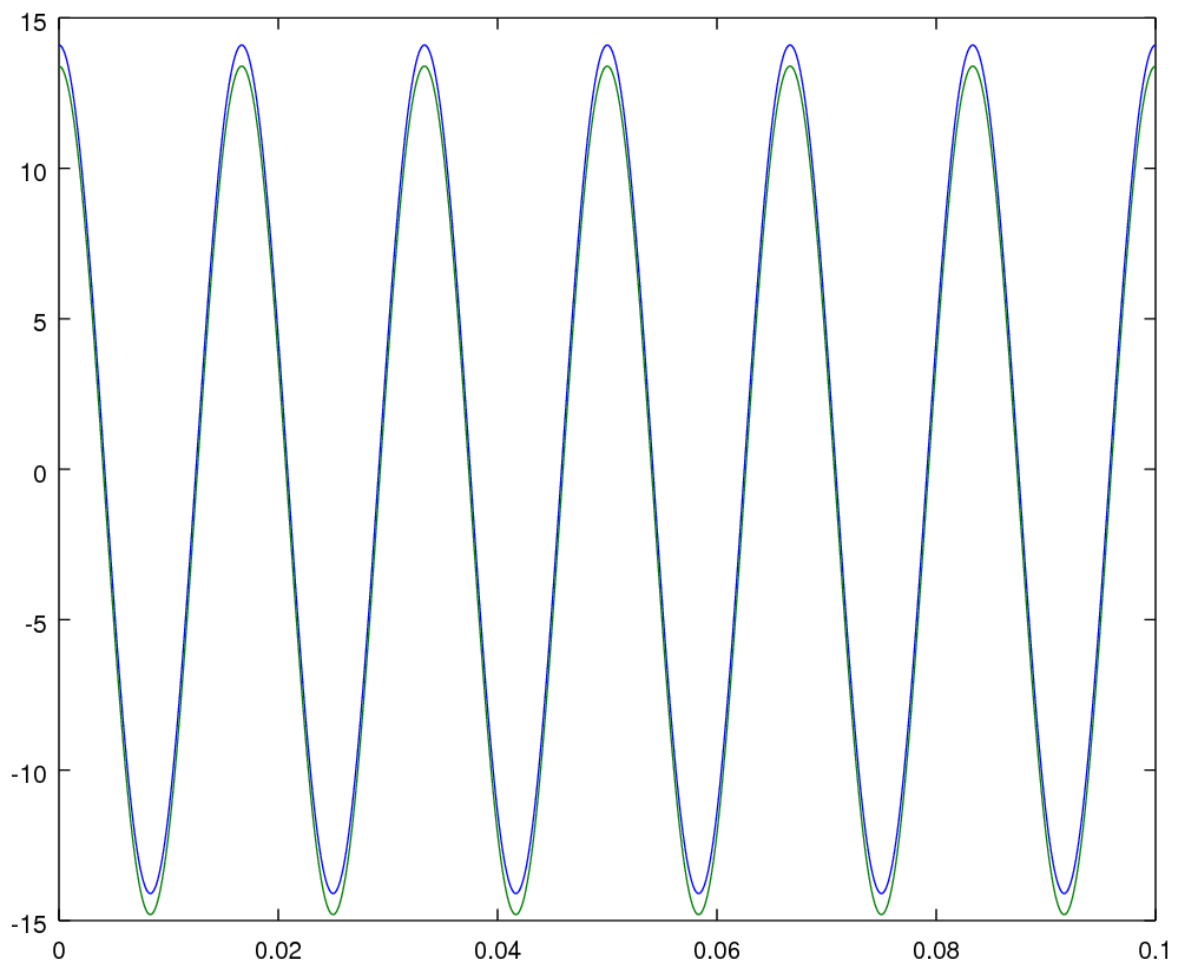


$\cos(4x)$

$$\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = 2\pi h \quad h = \frac{1}{\pi}$$

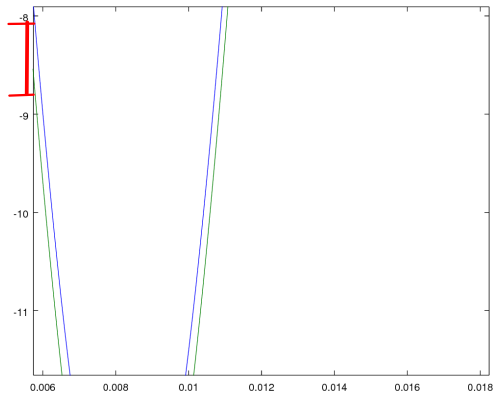




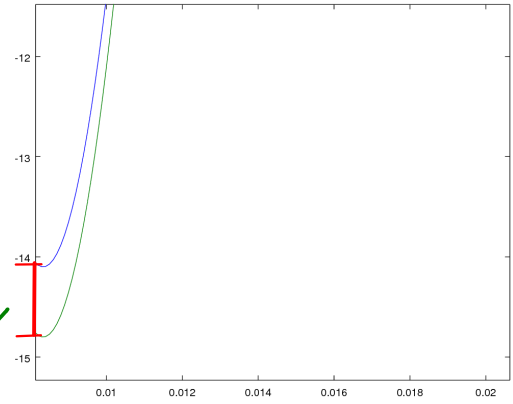


```
t = 0:0.0001:0.1;  
y = 14.1*cos(2*pi*60*t);  
plot(t, y)  
z = y - 0.7  
plot(t, y, t, z);
```

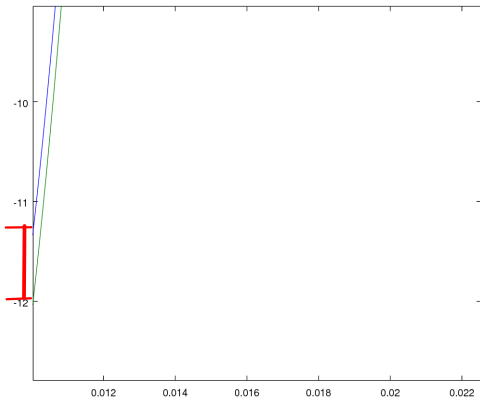
0.7V ↓



0.7V ↓



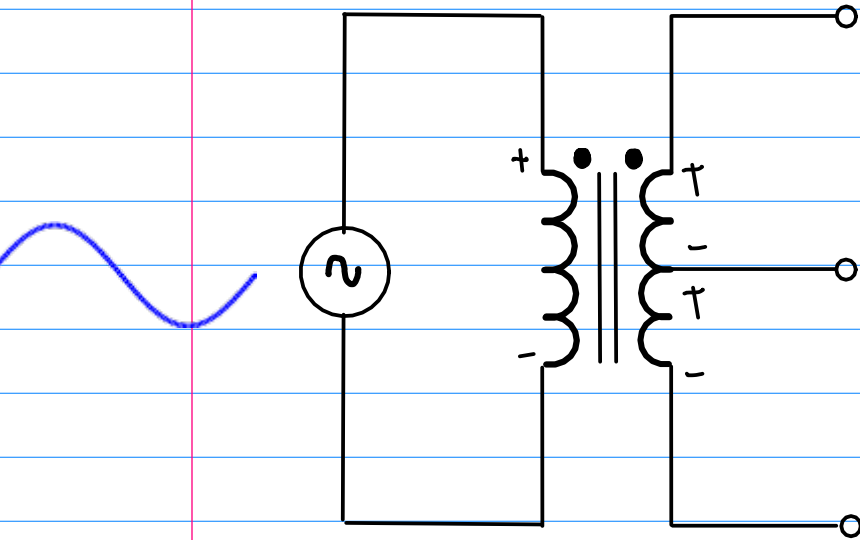
0.7V ↓



.

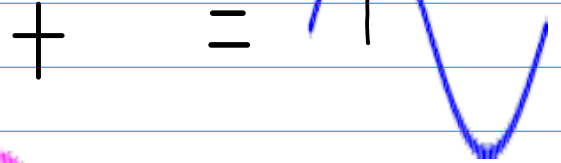
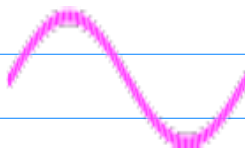
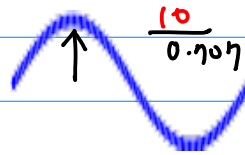
.

8



input voltage
of the half-wave
rectifier
= 10 Vrms

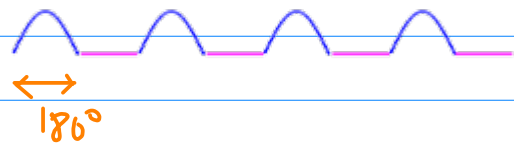
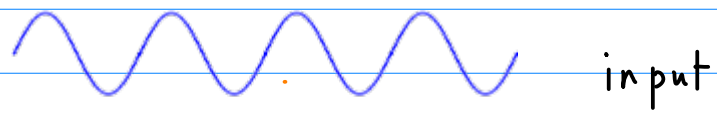
secondary voltage
= 20 Vrms



$$V_p = \frac{10}{0.707} = 14.1 \text{ V}$$

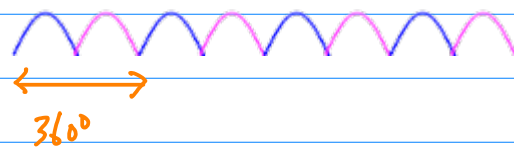
peak load voltage ↗

10



half wave rectifier

Current flows only for 180°

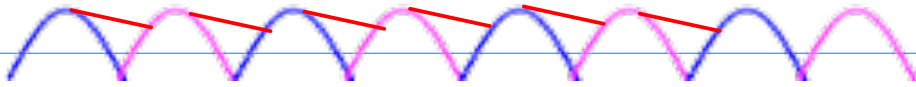
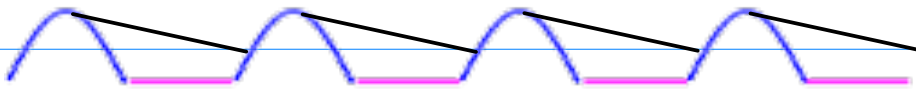
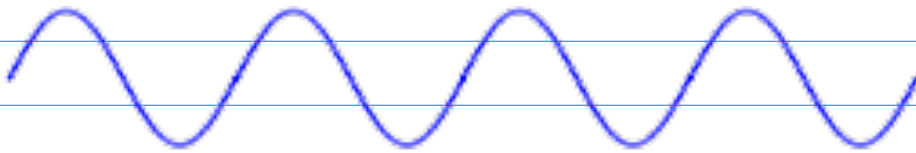


full wave rectifier

Current flows for 360°

#14

the same secondary voltage
and filter (R, C)



Full-wave



↓ small ripple

Half-wave



↓ large ripple

#15

Full-wave rectifier
center tap

